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Network centralities Interference and Robustness

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Abstract. We introduce the notions of centrality interference and centrality robustness, as measures of variation of centrality values when the structure of a network is modified by removing or adding individual nodes. As centrality analysis allows categorizing nodes according to their topological relevance in a network as centrality interference analysis allows understanding which parts of a network are mostly influenced by a node and, conversely, centrality robustness allows quantifying the functional dependency of a node from other nodes in the network.

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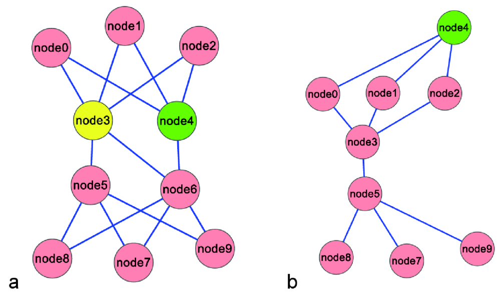
1. Introduction

One of the major goals of network science is the quantitative characterization of network structure and functionality with the purpose of inferring emergent properties of complex systems, abstracted as networks and represented as graphs [1]. The topological analysis approach allows understanding the functionality of networks through the analysis of their specific architecture. In this context, indexes of network centrality such as degree, eccentricity, closeness, betweenness, stress, centroid and radiality [7] are topological parameters allowing quantifying the topological relevance of single nodes in a network [2], [3], [5], [8]. We introduce the notion of interference and the Interference Cytoscape plugin [9], [4] to evaluate the topological effects of single or multiple nodes removal from a network. In this perspective, interference allows virtual node knock-out experiments: it is possible to remove one or more nodes from a network and analyze the consequences on network structure, by looking to the variations of the node centralities values. As the centrality value of a node is strictly dependent on the network structure and on the properties of other nodes in the network, the consequences of a node deletion are well captured by the variation on the centrality values of all the other nodes.

2. Interference and Robustness definition

Due to its importance and wide diffusion, we focus on node interference for betweenness centrality. The results can be also applied to other centrality indexes. Betweenness is defined as $Btw(G,n) = \sum_{s \neq n \in N} \sum_{t \neq n \in N} \frac{\sigma_{st}(n)}{\sigma_{st}}$ where σ_{st} is the number of shortest paths between s and t and $\sigma_{st}(n)$ is the number of shortest paths between s and t passing through the node n. Consider the network in figure 1. Node4 has high value of betweenness (15% of the total), since it connects the top of the network with the bottom (figure 1a). If we remove node6, node4 becomes a "pheripheral" node (figure 1b) and its betweenness value decrease (3.57%). At the same time, betweenness of node5 increases (from 15% to 42.86%) since it becomes the only node connecting the top and the bottom of the network (figure 1b). This is a case of interference of node6 with respect to node4 and node5, since the presence of node6 "interferes" with the betweenness values of these two nodes. Such variation can be measured with the interference notion defined above. **Interference definition:** All definitions consider connected networks (i.e. networks where each node is reachable from all the others), which remain connected even after node removal. This hypothesis is in agreement with results in attack tolerance for scale-free networks [6]. Given a network (a graph) $G = (N, E)$ where N is the set of nodes and E is the set of edges, the relative value of betweenness



Node	betwenness Network a	betwenness Network b	node6 Interference
node5	15.00	42.86	-27.86
node3	32.05	41.67	-9.61
node0	0.98	3.97	-2.99
node1	0.98	3.97	-2.99
node2	0.98	3.97	-2.99
node7	0.98	0.00	0.98
node8	0.98	0.00	0.98
node9	0.98	0.00	0.98
node4	15.00	3.57	11.43
node6	32.05		

Figure 1: **a.** Node3 and node4 connect the top of the network with the bottom. **b.** Node6 has been removed: node4 becomes a peripheral node, its betweenness decreases. The presence of node6 is important for node4 to play a central role (positive interference). At the same time, node5 and node3 becomes fundamental connections betweenn the top and the bottom. Their betweenness values increase. **right.** Betweenness and interference values expressed as percentage.

is obtained as $relBtw(G,n) = \frac{Btw(G,n)}{\sum_{j \in N} Btw(G,j)}$. It is the fraction of betweenness of each node with respect to the rest of the network. The interference of a node with respect to all the other nodes is detected by removing it from the network and recomputing the betweenness as follow. $G_{|i}$ is the network obtained from G removing node i and all its edges from the network. The $betwenness\ interference$ of a node i with respect to another node n in the network G is: $Int_{Btw}(i,n,G) = relBtw(G,n) - relBtw(G_{|i},n)$. The measure shows which fraction of the total betweenness value a node loses or gains with respect to the rest of the network when the node i is removed. The complete analysis of the network in the example is shown in the table of figure 1. The robustness notion is complementary to the interference one, and defined as $Rob_{Btw}(n,G) = \frac{1}{\max_{i \in N_{|n}}\{|Int_{Btw}(i,n,G)|\}}$. It is computed evaluating the interference of all the nodes in the network with respect to a single target node. This allows identifying the node or the group of nodes that more than others affect the functionality of a selected node, and if its role is dependent on any particular other node. In the example of figure 1 node3 has more "robustness" than node4, since there is no node removal that can consistently affects its betweenness value. On the contrary Node4 loses a consistent part of its betweenness when node6 is removed from the network. Node4 is not a "robust" node.

Positive and negative interference

As shown in the example, the interference value can be positive or negative. **Positive Interference.** If a node (A), upon removal from the network of a specific node (B), decreases its value for a certain centrality index, its interference value is positive. This means that this node (A), topologically speaking, takes advantage (is positively influenced) by the presence in the network of the node (B). Thus, “removal” of node (B) from the network, negatively affects the topological role of the node (A). This is called positive interference. It is the case of node6 with respect of node4 in the network of figure 1, where node4 is more “central” if node6 is part of the network. If node6 is removed node4 becomes a “peripheral node” and its betweenness value decreases: the presence of node6 positively ”interferes” with the betwenness of node4 (Interference value = 11.43, see table in figure 1). **Negative Interference.** If a node (A), upon removal from the network of a specific node (B), increases its value for a certain centrality index, its interference value is positive. This means that this node (A), topologically speaking, is disadvantaged (is negatively influenced) by the presence in the network of the node (B). Thus, “removal” of node (B) from the network, positively affects the topological role of node (A). This is called negative interference. It is the case of node6 with respect to node5 in the network of figure 1, where node5 is more “central” if node6 is removed from the network: the presence of node6 negatively “interferes” with the betwenness of node5 (Interference value = -27.86, see table in figure 1).

3. Potential applications and conclusions

The interference notion can be used to identify the area of interest of each node, and can model common situations where real nodes are removed or added from/to a physical network. In some cases, such as in social and financial networks, the structure of the network is naturally modified over time; in other cases this can be due to specific network changes: power grid failures, traffic jam or work in progress in a road network, temporary closure of an airport in an airline network and so on. In the case of Biological networks, one or more nodes (genes, proteins, metabolites) are possibly removed from the network because of gene deletion, pharmacological treatment or protein degradation. In the case of pharmacological treatment, one can potentially predict side effects of the drug by looking at the topological properties of nodes in a drug-treated network, meaning with that a network in which a drug-targeted node (protein) was removed. To inhibit a protein (for instance a kinases) corresponds to removing the node from the network. In the case of gene deletion which implies losing encoded proteins, it correspondto to a removal of one or more nodes from a protein network.

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