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Complexity control in a synchronized complex system

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Abstract. We numerically analyze the problem of how to drive a synchronized state in a complex system to other state with different complexity, keeping synchronization. The complex system used is obtained by synchronizing two identical chaotic Takens-Bogdanov sub-systems specially coupled to recover in the global system the symmetries of each oscillator. The global state is adjusted to have an initial synchronized hyperchaotic state (with two positive Lyapunov exponents). This work is an attempt, using small amplitude external signals, to drive the global system to other complex state keeping the synchronized state. The method used to overcome the problems that we had to select a useful frequency value for the driving signal will be discussed, together with a possible experiment in a thermo-convective flow for validating the results obtained.

Keywords: Synchronization, Hyperchaotic systems, Pre-turbulent convection.

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1. Introduction

Reduction in complexity of synchronized coupled systems has been frequently observed in numerical experiments controlling the coupling since the first works on confined systems [1] to large populations discussed in the frame of complex networks [2]. When a low dimensional system (3D) is analized, synchronization frequently supress chaos. In systems with higher dimensional dynamics, more recent works coupling hyperchaotic ODE have been reported to obtain synchronized states between identical systems without chaos suppression [4].

This work is focused on the driving with small amplitude signals two synchronized hyperchaotic systems, without destroy the synchronization between them. We use two Takens Bogdanov systems (with symmetry D4) coupled adequately to recover for the ensamble the symmetry of each equation system. Recently has been shown that a small amplitude harmonic [5] can be used to change the dynamical state in this system, if the frequency of the driving signal is properly chose. The first problem is that this frequency can not be obtained from the Fourier spectrum. In the cited work, a histogram constructed ad-hoc by observing the recurrent times in the output signal was used as a reference. As we remark forward, the histogram is not the result of the return times from a Poincare section, but it is simply a display of the more frequently visited times. The new state has been verified by obtaining the new Fourier spectrum and the attractor in the phase space. As the system has a Riddley bassin, a control of the output against a small change in the initial conditions was necessary to verify the independence of the global dynamics from initial conditions.

Finnally, we present among the new results, the autocorrelation in the output signal both for the “free-running” (without injection signal) and the controlled state. The larger autocorrelation time that can be observed in the controlled state, is a good indicator that, even if the system remains chaotic, becomes more coherent.

2. The dynamical system

Two identical oscillators TB1 and TB2 have been coupled to obtain the chaotic synchronized dynamical system that we will drive. Each one of them is a four dimensional Takens-Bogdanov’s system as it is shown in Eq. (1).

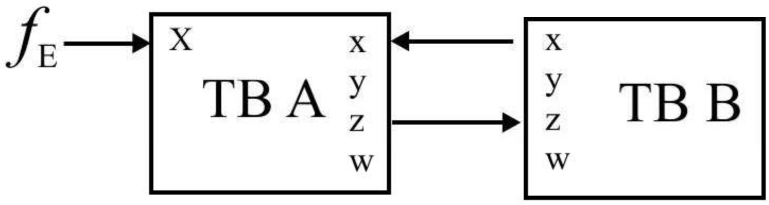

Figure 1: Interconnection scheme. The harmonic signal f_E is always injected to the variable x of the oscillator A

$$\dot{x} = y \tag{1}$$
$$\dot{y} = \mu x + x(a(x^2+z^2)+bz^2)$$
$$\dot{z} = w \tag{2}$$
$$\dot{w} = \mu z + z(a(x^2+z^2)+bx^2)$$

The symmetry properties of the dynamical equations are described by the symmetry group D_4 composed by reflection τ and rotation ρ (Eq. (3)):

$$\tau : \quad (x,y,z,w) \rightarrow (z,w,x,y) \tag{3}$$
$$\rho : \quad (x,y,z,w) \rightarrow (-x,-y,z,w)$$

These symmetries play an important role in the way that we have to connect A and B, as it is shown in Fig. 1.

Symmetrical coupling of two identical Takens-Bogdanov through one variable x has been presented in reference [4]. Synchronization regime has been analized using different coupling schemes (symmetric or asymmetric) and considering the coupling as a direct function of the error between both systems (acting as a feedback loop). Phase synchronization (PS) has been obtained using selected values for the parameters that must be fine-tuned to fit the Lyapunov exponent windows [6]. The synchronized regimes obtained in this case are not very stable and depend strongly on the coupling coefficient value.

Here we present a different approach. To obtain a robust synchronization manifold we recover for the coupled system the symmetry of each one oscillator. This is achieved by coupling the feedback on two different variables

(x and z). Internal symmetry of the equations couples variables by pairs (x and y to z and w). Introducing the coupling bidirectionally between A and B through x and z, we are constructing a closed "ring structure" that provides the robustness and Complete Synchronization (CS) is obtained. This was impossible to obtain in the case of a symmetrical coupling scheme based on only one variable [3]. The equations for the coupled system are now transformed in Eq. (4), where variables are named by the subscripts (A, B) corresponding to each original system. The coupled system is also hyperchaotic (having more than one positive Lyapunov exponent) and chaos is not completely suppressed by synchronization.

$$
\begin{aligned}
\dot{x}_A &= y_A + \varepsilon_x(x_B - x_A) + 0.01 x_{\mathrm{MAX}} \sin(2\pi f_E t) \\
\dot{y}_A &= \mu x_A + x_A(a(x_A^2 + z_A^2) + b z_A^2) \\
\dot{z}_A &= w_A \\
\dot{w}_A &= \mu z_A + z_A(a(x_A^2 + z_A^2) + b x_A^2) \\
\dot{x}_B &= y_B \\
\dot{y}_B &= \mu x_B + x_B(a(x_B^2 + z_B^2) + b z_B^2) \\
\dot{z}_B &= w_B + \varepsilon_z(z_A - z_B) \\
\dot{w}_B &= \mu z_B + z_B(a(x_B^2 + z_B^2) + b x_B^2)
\end{aligned}
\tag{4}
$$

The coupling terms $\varepsilon_x(x_B - x_A)$ and $\varepsilon_z(z_A - z_B)$ can be interpreted as the feedback signals between both systems. These factors are equal to zero when complete synchronization is achieved and both systems reproduce the same trajectory on the synchronous manifold separately whitout any feedback between them. This kind of coupling extends the inner symmetries of each equation [4]. system to the coupled one which has a higher dimension. The results can be seen in Fig. 2.

Variables (x, y, z, w) in A are completely synchronized to (x, y, z, w) in B. This mean that x_A is complete synchronized to x_B and so on. Temporal signals displaying the synchronized state will be shown in the next paragraph together with effects of a driving signal $f_E = 0.01 x_{\mathrm{MAX}} \sin(2\pi f_E t)$.

Fig. 2 (a) displays the Fourier spectrum for one variable (variable x_A) and a low frequencies detail appears in Fig. 2 (b). This figure shows a characteristic spectrum of a chaotic signal. Synchronization can be appreciated in Fig. 2 (c), where the synchronization error function against time goes to zero after a short transient.

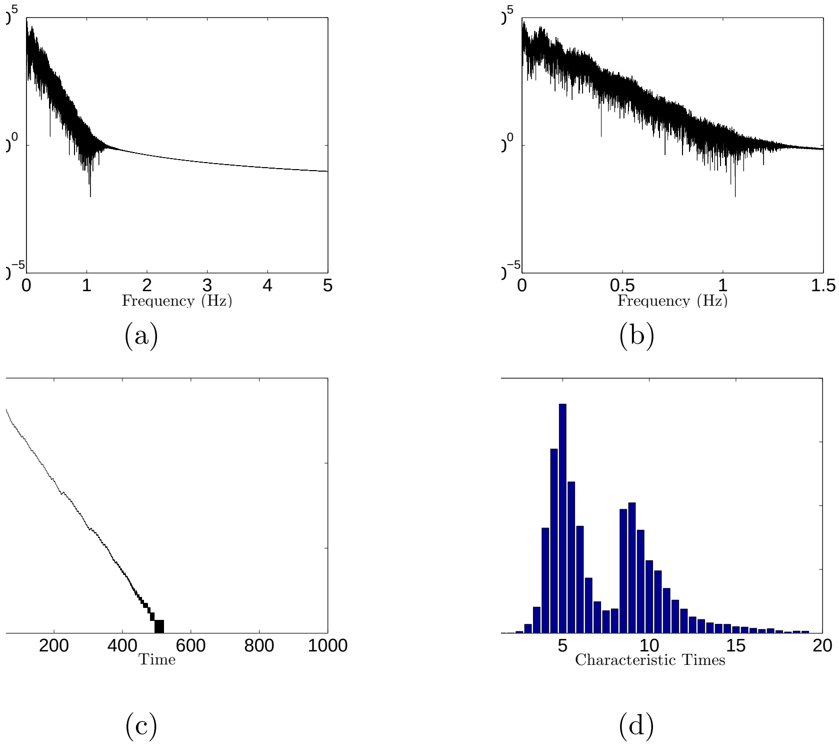

Figure 2: Figure (a) Fourier spectrum for variable x_1. (b) Zoom on low frequencies. (c) Error function for synchronization in logarithmic scale. The function going to zero against time illustrate the necessary condition for CS. (d) Histogram of recurrent times (in seconds) that allow us to obtain the system characteristic times.

3. Driving the coupled system

This information, (that can be easily obtained from an experiment), is not sufficient if we need to choose a frequency for driving the system. As in many other cases (like in the Rössler attractor for the "Fünnel" parameters), here also is not possible to construct a Poincaré section, and under these restrictions we cannot define an "analytical phase" [1]. To overcome this problem we constructed an histogram considering the period between two neighbours maximum (or minimum [5]), obtained from a very long data file of the output signal. The amplitud in the histogram represents at each time value, the frequency (number of times) that this period value appears in the output signal, being the highest peak the period that most frequently appears. Resolution in time can be controled by choosing the width of the time interval to count, and in amplitud, it is one count over the total number of counts. A typical histogram constructed from the output signal of the coupled system is shown in Fig. 2 (d). From this histogram, with periods distributed around two main peaks, we can obtain the most recurrent values of the frequencies in the system.

4. Results and Conclusions

The histogram has been used to localize the region of frequencies that should be applied. To determine which of the frecuencies in the interval is the best to act on the system, we reconstruct histograms injecting signals with several frecuencies between the two peaks. Clearly the central frequency gives a state with a different dynamic (figure 3). To check it accurately we obtained the new Forier spectrum, the new phase diagram and the autocorrelation for the variable x. The results for autocorrelation are presented in figure 4. The driven state display a longer correlation (zero crossing) for the drived system. It is a clear indication that we obtained a more ordered system with a lower complexity and a higher coherence.

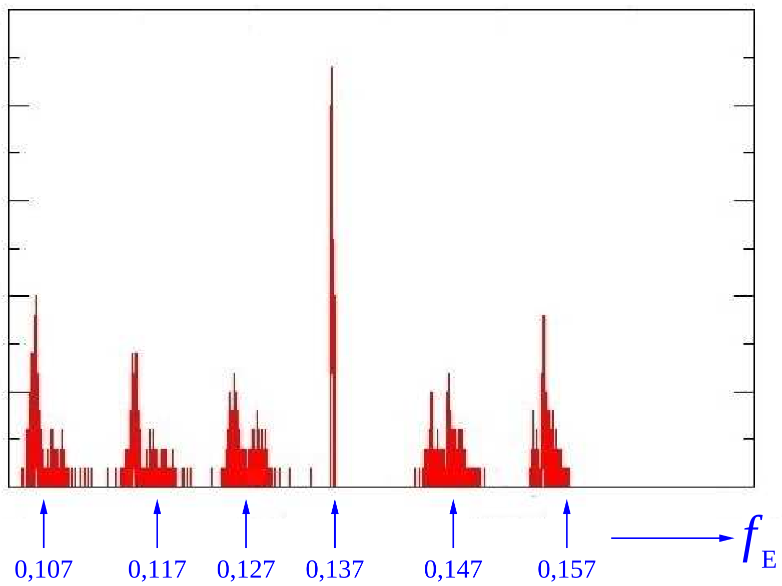

Figure 3: Sequence of histograms for the system driven with a harmonic signal with different frequencies f_E injected to the variable x of the oscillator A

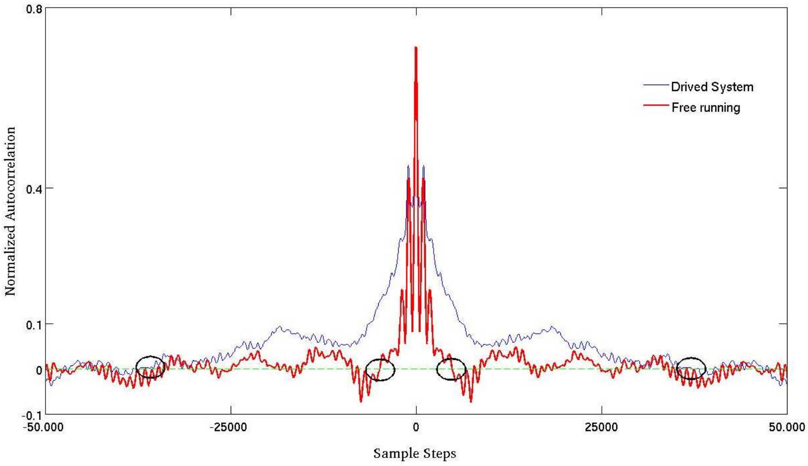

Figure 4: Comparison of autocorrelation in the variable x obtained from the free running (red) and the driven state (blue)

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