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Analysis of chaos existence in european stock markets

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Abstract. Interest in nonlinear dynamics, especially deterministic chaotic dynamics, has grown in both the financial papers and the academic literature. This has come about because the frequency of large moves in stock markets is greater than would be expected under a gaussian distribution.

This paper tests the presence of chaos in some stock market indexes: MDAX, DAX, DOW JONES, EURO STOXX 50, Euronext 100 and others. Several parameters are measurement in the data series: **H**urst **C**oefficients (HC), **F**ast **F**ourier **T**ransform (FFT), **R**ecurrence **Q**uantification **A**nalysis (RQA), **L**yapunov **E**xponents (LE) and **C**orrelation **D**imension (CD). Due to an effective forecasting model would reduce risks, assist in planning and decision making, serie temporal evolution is predicted additionally.

HC characterizes the persistence behavior in a serie: a value equal to 0.5 indicates that the sequence is either random or uncorrelated, a value in the range $0.5 < H < 1$ corresponds to power-law correlations and the range $0 < H < 0.5$ reflects the presence of anti-correlations. We study the frecuency spectrum ($S(\nu) = \sqrt{|FFT|}$ and the DC by means of Martelli's Method [1]. We also investigate the presence of deterministic dependencies using recurrence graphics and RQA [2]. This research also obtain the LE by means of Rosenstein Mechanism [3].

Keywords: Fast Fourier Transform, Recurrence Quantification Analysis, Correlation Dimension , Hurst Coefficient, Lyapunov Exponent

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1. Introduction

Our research studies several european stock market indexes: DOW JONES, EURONET 100, MDAX and others. Last Value and the Change Magnitude of these indexes are collected from 2011 until the present year; some mathematical parameters are analized to identify the chaos presence in the data set.

2. Analyzing the European Stock Markets

DISCRETE FOURIER TRANSFORM

Given a sequence of N samples $X(n)$, indexed by $n = 0 \cdots N-1$, its **D**iscrete **F**ourier **T**ransform (DFT) is defined as:

$$F(k) = \frac{\Sigma_{k=0}^{N-1} x(k) e^{-j2\pi nk/N}}{\sqrt{N}} \tag{1}$$

$F(k)$: Fourier Coefficients,

$|F(k)|$: Power Spectrum.

Power Spectrum ($S(\nu)$, ($|F(k)|$ as k function) determines the spectral content of a time serie from a finite set of measurements. We show the Power Spectrum for the Change Series of DOW JONES, EURONET 100 and MDAX Indexes in Fig.1 and Fig.2, where no clear periodic components are distinguished and a frequence continuity is present, power of the signal is not constant.

RECURRENCE QUANTIFICATION ANALYSIS

Takens' Theorem [4] states that it is possible to recreate a topologically equivalent picture of the original multidimensional system behaviour, using the time series of a single observable variable, by means of the method of time delays: for scalar series $\{x_t\}_{t=1}^{T}$ we build up the embedded vectors:

$$X_i^m = (X_i, X_{i+\tau}, X_{i+2\tau}, ..., X_{i+(m-1)\tau}) \tag{2}$$

The set of all embedded vectors, $i = 1, ..., T-(m-1)\tau$, is a trajectory in R^m, where m is the embedding dimension and τ is the time delay. The sequence of embedded vectors recreates the original dynamics only if parameters m and τ are properly selected by using methods like **F**alse **N**earest **N**eighbours (FNN) (for m estimation) and **M**utual **I**nformation (for τ calculation). The choice of m must assure that $m > 2d+1$, where d is the original systems dimension. Euclidean Distances D

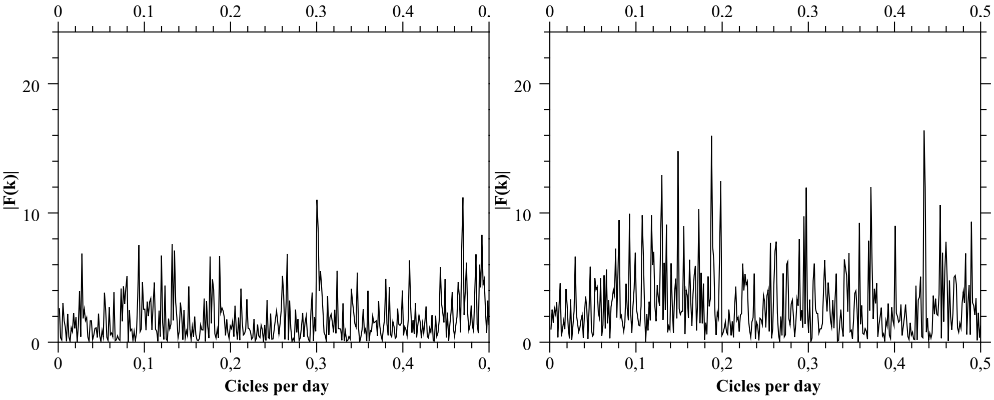

Figure 1: Left side: Power Spectrum of Change Serie for DOW JONES Index. Right side: Power Spectrum of Change Serie for EURONET 100 Index

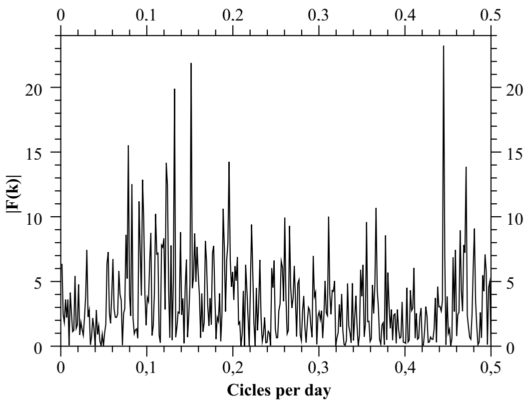

Figure 2: Power Spectrum of Change Serie for MDAX Index

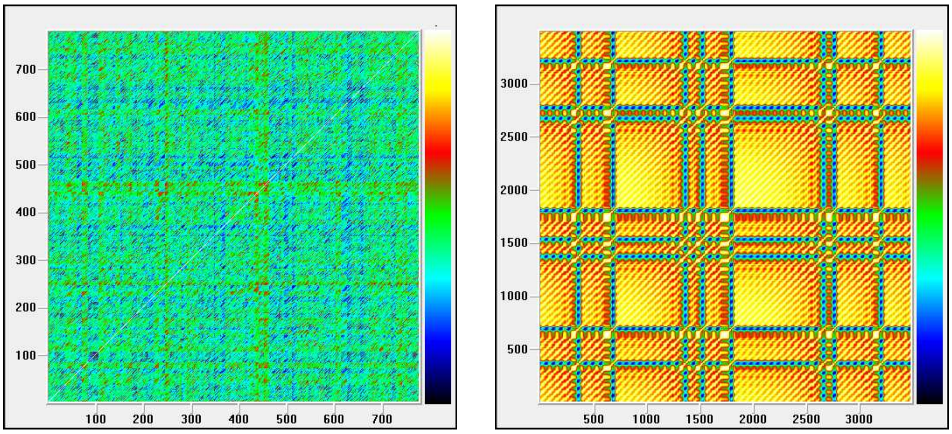

Figure 3: Left side: RP for a random serie. Right side: RP for Lorentz' Attractor

are calculated between all pairs $(\hat{U},\hat{V})$ of embedded vectors.

A **R**ecurrence **P**lot (RP) is a graphical representation of the distances matrix D_{UV}, by colouring the pixel located at coordinates $(\hat{U},\hat{V})$ that correspond to a distance value between $\hat{U}$ and $\hat{V}$ vectors lower than a predetermined cutoff value ξ. The graph is symmetric $(D_{UV}=D_{VU})$, and that the main diagonal is always coulored $(D_{UV}=0,\hat{U}=\hat{V})$. The resulting RP is a colored-coded matrix, where each D_{UV} entry is mapped to colors from the pre-defined color map and is displayed as a colored pixel in the corresponding place.

The estimated distances (D) represent the evolution of the orbits in the Phase Space. A RP of a chaotic system will be more organized and will alternate the colour intensities (from stronger to lower strength), which denotes contraction and expansion of the system in some specific moments. A random systems show a uniform structure and low alternation of colour intensity, which means that the calculated distances (D) are mostly homogeneous.

We carry out the RQA [2]. Fig. 3 shows the RP for a Random Serie and for Lorentz' Attractor.

In the Last Value Series of the studied indexes the RP shows a strong trend and curve lines due to periodicity. This is shown in the Fig. 4, Fig. 5, and Fig. 6. In the Change Series, we observe determinism signs (more uniformity) and the existence of unstable periodic orbits denoted by the presence of horizontal lines, which is depicted in Fig. 7 and Fig. 8.

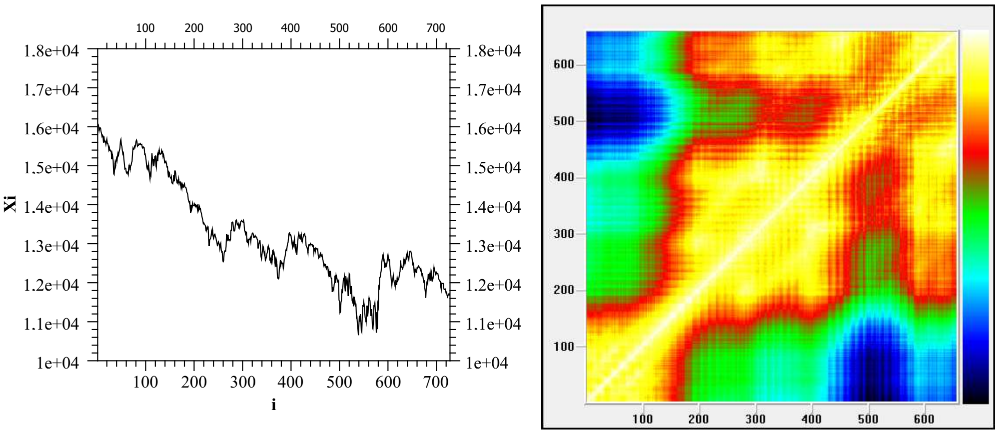

Figure 4: Left side: Last Value Serie for DOW JONES Index. Right side: RP for Last Value Serie of DOW JONES Index (m=6; τ=14)

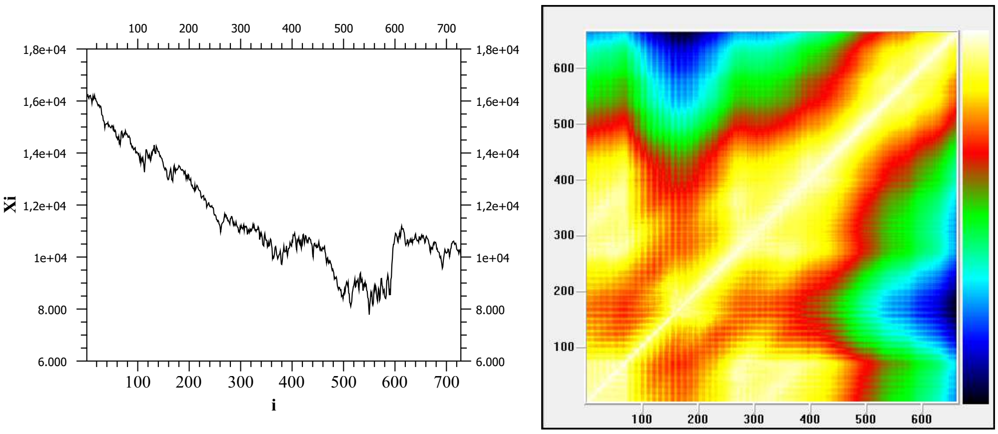

Figure 5: Left side: Last Value Serie for MDAX Index. Riht side: RP for Last Value Serie of MDAX Index (m=8; τ=11)

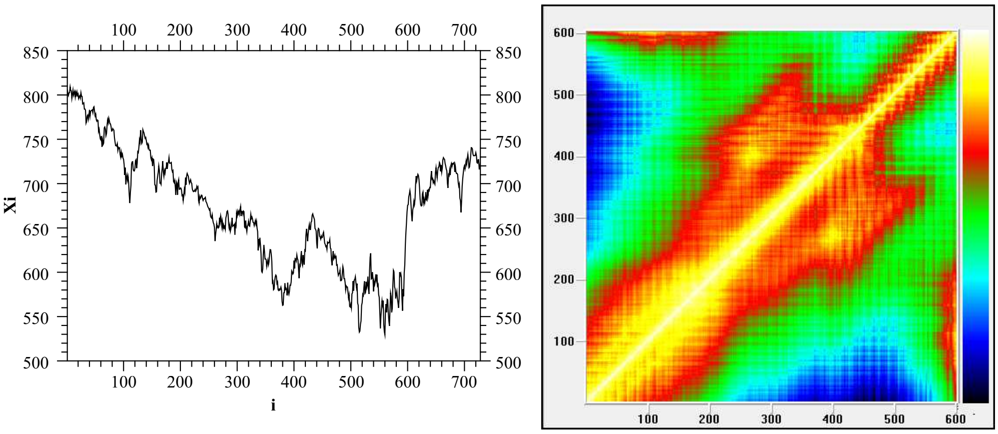

Figure 6: Left side: Last Value Serie for EURONET 100 Index. Right side: RP for Last Value Serie of EURONET 100 Index (m=10; τ=16)

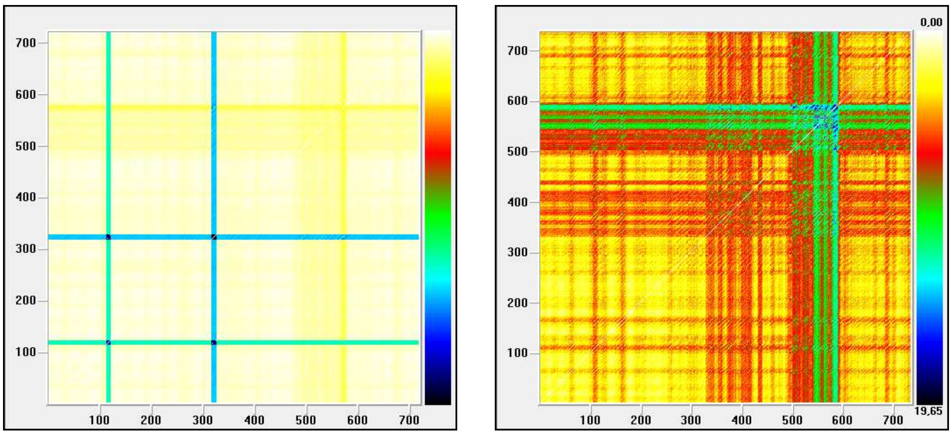

Figure 7: Left side: RP for Change Values of DOW JONES Index (m=9; τ=1). Right side: RP for Change Values of MDAX Index (m=9; τ=1)

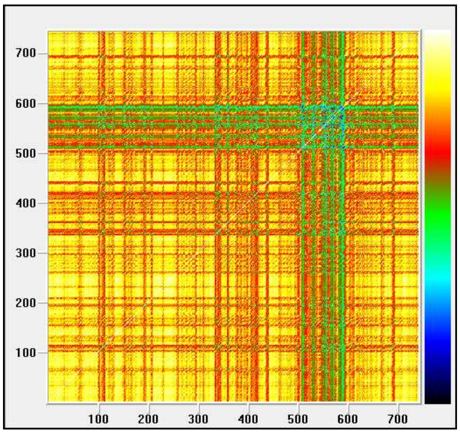

Figure 8: RP for Change Values of EURONET 100 Index (m=4; τ=1)

HURST COEFFICIENT

Hurst **C**oefficients (HC) can be defined as:

$$E\{\frac{R(n)}{\sigma(n)}\} = C_n^H; n \to \infty \tag{3}$$

where:

n: Number of elements in a time series,

$R(n)$: Range of a time serie,

$\sigma(n)$: Standard Deviation of $R(n)$,

$E(v)$: Expected Value of the random variable v: $E(v) = \Sigma_{i=1}^{k} v_i p_i$; where v can take the value v_i with probability p_i.

C: Constant.

HC calculation requires to estimate the dependence of the rescaled range on n observations. A time serie of full length N is divided into a number of shorter time series of length $n = N, N/2, N/4, \cdots$. Then, for a time serie of length n, the rescaled range is obtained as follows:

1. To estimate the average: $< a >= \frac{1}{n}\Sigma_{i=1}^{n} X_i$,
2. To build a mean-adjusted series: $Y_t = X_t - < a >$ for $t = 1, 2, \cdots n$
3. To calculate the cumulative deviate series Z: $Z_t = \Sigma_{i=1}^{t} Y_i$ for $t = 1, 2, \cdots n$,

4. To estimate R: $R(n) = max\{z_1 \cdots z_n\} - min\{z_1 \cdots z_n\}$,
5. To compute $\sigma(n)$: $\sqrt{\sigma_{i=1}^{n}(X_i - < a >)^2/n}$,
6. To calculate $R(n)/\sigma(n)$ and average over all the partial time series of length n.

HC is estimated by fitting the power law $E\{\frac{R(n)}{\sigma(n)}\} = C_n^H; n \to \infty$ to the data. This magnitude can be:

$HC = 0.5$ (White Noise): series elements are completely random and independent, there is not correlation between the values.

$0.5 < HC \leq 1$: serie values show a persistent or correlated process. Every event that happens today will impact the future, modifications in a day are correlated with all future changes. If $H = 1$ the serie has a deterministic behaviour (Black Noise).

$0 \leq HC < 0.5$: value series show a non persistence or non correlated behaviour , they tend to return to the origin place (Rose Noise).

Table 1 show the HC for Last Value Series and Change Series of DOW JONES, EURONET 100 and MDAX Indexes. We can observe that EURONET 100 and MDAX indexes have a persistent behaviour.

Table 1: HC for stock market indexes: DOW JONES, EURONET 100 and MDAX

Stock market index	Serie	*HC*
DOW JONES	Change	0,47210
	Last value	0,50532
EURONET 100	Change	0,63321
	Last value	0.87825
MDAX	Change	0,63876
	Last value	0.9785

CORRELATION DIMENSION

Correlation Dimension $C(r)$ is the probability that a pair of points is located at a distance r from each other in an attractor. This magnitude can be calculated as:

$s(i, j) =| X_i - X_j |$: distances between a pair of elements in the set of N points.

$C(r) = \frac{1}{N^2}\{\, number\ of\ pairs\ (i, j)\ with\ \ s(i, j) < r\}$

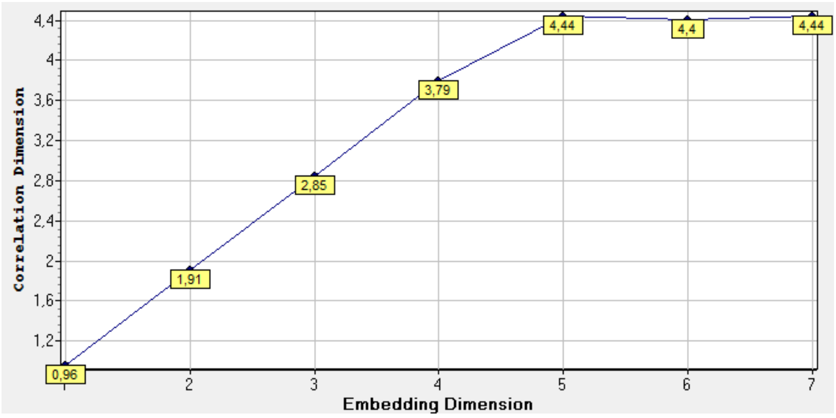

Figure 9: Dimension Correlation for Change Serie of MDAX Index (τ=1)

$C(r)\ follows\ a\ power\ law: C(r) = kr^D$ and can be written as:

$$C(r) = lim_{N\to\infty}\frac{\Sigma_{j=1}^{N}\Sigma_{i=j+1}^{N}\emptyset(r-\mid X_i - X_j\mid)}{N^2} \tag{4}$$

$$\emptyset(r-\mid X_i - X_j\mid) = \{_0^1 \quad {}_{0>(r-|X_i-X_j|)}^{0\leq(r-|X_i-X_j|)}\} \tag{5}$$

We calculate the DC by means of Martelli's Method [1].

In Fig. 9 we show $C(r)$ for the Change Serie of MDAX Index, it is possible to observe that the slope is stabilized with increasing insertion dimension which is a common feature for all studies indexes. This chracteristic appears in the chaotic systems.

LYAPUNOV EXPONENT

Lyapunov Exponent of a dynamical system characterizes the rate of separation of infinitesimally close trajectories. Quantitatively, two trajectories in phase space with initial separation δZ_0 diverge at a rate given by: $|\delta Z_t| \simeq e^{-\lambda t}|\delta Z_0|$; where λ is the Lyapunov Exponent. For discrete time system $x_{n+1} = f(x_n)$, the Maximal Lyapunov Exponent can be defined as follows:

$$\lambda = \frac{1}{n}\Sigma_{i=0}^{n-1} ln|f^{'}(x_n)| \tag{6}$$

We calculated λ by means of Rosenstein's Method [3] and we obtained that all studied series had a positive λ. This is a typical property in a complex system.

3. Conclusions

This paper has investigated whether some european stock market indexes are characterized by a nonlinear dependence which has been confirmed through different parameters for some of them.

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